刚体转动	一般刚体运动
$R \in SO(3)$:3×3矩阵 $R^TR = I, detR = 1$	$T \in SE(3)$:4×4矩阵 $T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}, R \in SO(3), P \in \mathbb{R}^3$
$R^{-1} = R^T$	$T^{-1} = \begin{bmatrix} R^T & -R^Tp \\ 0 & 1 \end{bmatrix}$
坐标系变换: $R_{ab}R_{bc} = R_{ac}, R_{ab}p_b = p_a$	坐标系变换: $T_{ab}T_{bc} = T_{ac}, T_{ab}p_b = p_a$
旋转坐标系$\{b\}$: $R = Rot(\hat{\omega}, \theta)$ $R_{sb'} = RR_{sb}$(绕轴线$\hat{\omega}_s = \hat{\omega}$转动$\theta$) $R_{sb''} = R_{sb}R$(绕轴线$\hat{\omega}_b = \hat{\omega}$转动$\theta$)	旋转坐标系$\{b\}$: $T = \begin{bmatrix} Rot(\hat{\omega}, \theta) & p \\ 0 & 1 \end{bmatrix}$ $T_{sb'} = TT_{sb}$(先绕轴线$\hat{\omega}_s = \hat{\omega}$转动$\theta$,再相对于$\{s\}$移动$\{b\}$原点距离$p$) $T_{sb''} = T_{sb}T$(先相对于$\{b\}$移动$\{b\}$原点距离p,再绕新的坐标系中的轴线$\hat{\omega}$转动θ)
单位转轴$\hat{\omega} \in \mathbb{R}^3$ 其中,$\|\|\hat{\omega}\|\| = 1$	单位螺旋轴$\mathcal{S} = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$ 其中,①$\|\|\hat{\omega}\|\| = 1$或②$\omega = 0, \|\|v\|\| = 1$ 对于具有有限节距h的螺旋轴$\{q, \hat{s}, h\}$ $\mathcal{S} = \begin{bmatrix} \omega \\ v \end{bmatrix} = \begin{bmatrix} \hat{s} \\ -\hat{s} \times q + h\hat{s} \end{bmatrix}$
角速度$\omega = \hat{\omega}\dot{\theta}$	运动旋量$\mathcal{V} = \mathcal{S}\dot{\theta}$
对于三维向量$\omega \in \mathbb{R}^3$, $[\omega] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \in so(3)$ 对于$\omega, x \in \mathbb{R}^3$,$R \in SO(3)$,$[\omega] = -[\omega]^T$, $[\omega]x = -[x]\omega$,$[\omega][x] = ([x][\omega])^T$, $R[\omega]R^T = [R\omega]$	对于$\mathcal{V} = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$,$[\mathcal{V}] = \begin{bmatrix} [\omega] & v \\ 0 & 0 \end{bmatrix} \in se(3)$ (ω, v)可描述运动旋量$\mathcal{V}$或单位螺旋轴$\mathcal{S}$,取决于上下文
$\dot{R}R^{-1} = [\omega_s]$,$R^{-1}\dot{R} = [\omega_s]$	$\dot{T}T^{-1} = [\mathcal{V}_s]$,$T^{-1}\dot{T} = [\mathcal{V}_s]$ $[Ad_T] = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \in \mathbb{R}^{6\times 6}$ $[Ad_T]^{-1} = [Ad_{T^{-1}}]$,$[Ad_{T_1}][Ad_{T_2}] = [Ad_{T_1T_2}]$

刚体转动	一般刚体转动
坐标系变换: $\hat{\omega}_a = R_{ab}\hat{\omega}_b,\ \omega_a = R_{ab}\omega_b$	坐标系变换: $\mathcal{S}_a = [Ad_{T_{ab}}]\mathcal{S}_b, \mathcal{V}_a = [Ad_{T_{ab}}]\mathcal{V}_b$
$R \in SO(3)$的指数坐标:$\hat{\omega}\theta \in \mathbb{R}^3$ $exp : [\hat{\omega}]\theta \in so(3) \to R \in SO(3)$ $R = Rot(\hat{\omega}\theta) = e^{[\hat{\omega}]\theta} = I + \sin\theta[\hat{\omega}] + (1 - \cos\theta)[\hat{\omega}]^2$ $log : R \in SO(3) \to [\hat{\omega}]\theta \in so(3)$ 相关算法参考课件3.2.3	$T \in SE(3)$的指数坐标:$\mathcal{S}\theta \in \mathbb{R}^6$ $exp : [\mathcal{S}]\theta \in se(3) \to T \in SE(3)$ $T = e^{[\mathcal{S}]\theta} = \begin{bmatrix} e^{[\omega]\theta} & temp \\ 0 & 1 \end{bmatrix}$ $\text{temp} = (I\theta + (1 - \cos\theta)[\omega] + (\theta - \sin\theta)[\omega]^2)v$ $log : T \in SE(3) \to exp : [\mathcal{S}]\theta \in se(3)$ 相关算法参考课件3.3.3
力矩的坐标变换: $m_a = R_{ab}m_b$	力矩的坐标变换: $\mathcal{F}_a = (m_a, f_a) = [Ad_{T_{ba}}]^T\mathcal{F}_b$